



On the analysis of buoyancy-driven hydromagnetic second grade fluid flows in the presence of suction and blowing

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ABSTRACT

This paper is concerned with the investigation of steady two-dimensional buoyancy-driven hydromagnetic mixed convection boundary layer flow of second grade fluid over a nonlinear stretching sheet in the presence of suction and blowing. Well-established similarity transformations are efficiently used to reduce the dimensional partial differential equations governing the flow and thermal fields into a set of non-dimensional ordinary differential equations with the necessary conditions. The standard bvp4c MATLAB solver is effectively used to solve the fluid flow equations to get the numerical solutions in terms of velocity and temperature fields. Numerical results are obtained for a different set of physical governing parameters and their behaviour is described through graphs and tables. The second grade fluid parameter enhances the velocity field whereas the magnetic parameter suppress the velocity field in the flow region. The temperature field is magnified for increasing values of the mixed convection and heat generation parameter.

Keywords: Second grade fluid, mixed convection, suction/blowing, heat generation/absorption and Eckert number.

1.0. Introduction

Interest in the boundary layer flows over a stretching surface has increased substantially during the last few years due to its several industrial applications. The study of boundary layer behaviour of fluids with stretching of elastic sheet from a fixed point with or without varying velocity has applications in many manufacturing processes, for instance paper making, polymer processing, food processing, oil recovery etc. Extensive research on this topic has been undertaken since the pioneering work of Sakiadis (1961) (flow over a uniformly moving continuous surface). In view of this, Crane (1970) obtained an exact analytical solution for the

boundary layer flow when the stretching velocity is proportional to the distance from the origin.

Non-Newtonian fluids differ from Newtonian fluids in viscosity, which is defined in terms of shear or tensile stresses; it is the fluid's ability to resist gradual deformation. The relationship between shear rate and shear stress can be time dependent (Bar-Meir, 2009). Some non-Newtonian fluids exhibits plastic or elastic behavior, examples include paint, ketchup, soup, blood and toothpaste. The boundary layer flow of a non-Newtonian fluid past a continuously moving solid surface/sheet is a very special and significant type of flow

existing in many industrial and engineering processes. The main branch of modern fluid dynamics is the study of heat transfer from solid geometry to an ambient fluid due to the continuous stretching of the sheet/surface through the free stream. These types of heat transfer studies have a large number of significant technological applications in various branches of science and engineering, particularly in the field of aerodynamics, gas dynamics and chemical engineering. For instance, processes such as polymer extrusion and metallurgical process are studied under the branch of chemical engineering, and they comprise the refrigeration of molten fluid, which is being stretched into the cooling system (Basha *et al.*, 2019). Hence, the need for study of non-Newtonian fluids with particular emphasizes on the second grade fluid is of great importance.

Investigation on boundary layer behavior of a second-grade fluid over a continuously stretching surface finds applications in engineering processes. Some of these applications include polymer extrusion, drawing of plastic films and wires, glass fiber and paper production, manufacture of foods, crystal growing, liquid films in condensation process, etc. (Prasad *et al.*, 2010a). The ever increasing applications in these industrial processes have led to a renewed interest in the study of second grade fluid flow and heat transfer over a stretching sheet. For this purpose, Mamaloukas *et al.* (2007) demonstrated the likeness of the free-parameter method and the separation of variables method to determine the exact solution for the problem of boundary layer flow of an incompressible viscoelastic second grade fluid along a stretching sheet. Hayat *et al.* (2008) applied an analytic technique, namely the Homotopy analysis method (HAM), to study the steady mixed convection in two-dimensional stagnation flows of a second-grade fluid around a heated surface with the wall temperature varying linearly with the distance from the stagnation point. In the

same vein, Hayat *et al.* (2016) also examined the magnetohydrodynamic flow of second grade nanomaterial by a nonlinear stretched surface. Heat and mass transfer of an electrically conducting second grade MHD fluid past a semi-infinite stretching sheet with convective surface heat flux using Runge–Kutta sixth order integration scheme is studied by Das *et al.* (2016). Their analysis explores that the rate of heat transfer enhances with increasing the values of the surface convection parameter. Recently, Fetecau *et al.* (2020) analytically study hydromagnetic flow of second grade fluids with time fractional derivatives without singular kernel over an infinite moving plate and found out that the flow of fractional order fluids is faster as compared to the ordinary fluids at smaller time steps.

We know that the properties of the final product in industries depend greatly on the rate of cooling involved in manufacturing processes. It would be beneficial to have a controlled cooling system for these processes. An electrically conducting polymeric liquid seems to be a good candidate for some industrial applications such as in polymer technology and metallurgy because the flow can be regulated by external means through a magnetic field. The applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet. Several analyses have been carried out recently to investigate the MHD boundary layer flow and heat transfer characteristics of a viscoelastic second grade fluid past a stretching surface.

Hsiao (2010) investigated MHD flow of a second grade fluid near stretching sheet with electric and magnetic dissipation and non-uniform heat source/sink. His investigation revealed that the electric dissipation parameter increased the heat transfer at the wall while the magnetic dissipation parameter decreased it. Hayat *et al.* (2012) analyzed the impact of induced magnetic

field on second-grade nanofluid flow past a convectively heated stretching sheet and it has been observed that larger values of viscoelastic parameter (second grade fluid parameter) results in an increment in temperature distribution. In the analysis of Datti et al. (2016), the effects of thermal radiation and temperature-dependent thermal conductivity on MHD viscoelastic flow have been examined. Abbas et al. (2008) carried out an analysis to study the unsteady MHD two-dimensional boundary layer flow of a second-grade viscoelastic fluid over an oscillatory stretching surface. The flow is induced due to an infinite elastic sheet, which is stretched back and forth in its own plane.

Mixed convection heat transfer arises in many transport processes both in natural and engineering applications (Patil et al., 2017). Such processes occur when the effects of buoyancy forces in forced convection or the effects of forced flow in natural convection become much more significant. Hence, the characteristics of heat transfer in a mixed convection flow over a stretching surface has important real world applications in polymer technology and metallurgy. The fluid mechanical properties desired for the outcome of such a process would mainly depend on the rate of cooling and stretching rate. So, one has to pay considerable attention in knowing the heat transfer characteristics of the stretching sheet as well. Motivated by this process, Prasad et al. (2010b) investigated mixed convection flow over a nonlinear stretching surface with variable fluid characteristics. Pal and Mondal (2011) discussed MHD non-Darcian mixed convection heat and mass transfer over a non-linear stretching sheet. Similarly, Kumar et al. (2017) studied the effect of nonlinear thermal radiation on double-diffusive mixed convection boundary layer flow of viscoelastic nanofluid over a stretching sheet and found out that the effect of nanoparticle volume fraction and nonlinear thermal radiation stabilizes the thermal boundary layer

growth. Turkyilmazoglu (2013) constructed the analytical solution of mixed convection flow of MHD viscoelastic fluid over a permeable stretching surface. Furthermore, Alsaadi et al. (2013) studied the mixed convection flow of second grade fluid bounded by a permeable stretching surface with Soret and Dufour effects.

Suction or blowing of a fluid through the stretching surface can significantly change the flow field and therefore, affects the heat transfer rate from the surface. In general, suction and blowing tends to be opposite, suction increases the skin friction coefficients and heat transfer rate, whereas blowing has the opposite tendency (Al-Sanea, 2004). Blowing or withdrawal of fluid through a permeable bounding heated or cooled surface is of general interest in practical problems involving boundary layer control applications such as film cooling, polymer fiber coating and coating of wires (Wang, 2011). The process of suction and blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers and thermal oil recovery. Suction is applied to chemical processes in order to remove reactants. Blowing is used to add reactants, cool the surface, prevent corrosion or scaling, and reduce the drag. Vajravelu and Hadjinicolaou (1993) examined the heat transfer characteristics in the laminar boundary layer flow of viscous fluid over a linearly stretching surface with variable wall temperature subject to suction or blowing (injection). Ali and Al-Yousef (2002) investigated the effect of surface suction/injection with the form of general power function for stretching velocity of the wall. Sandeep and Sulochana (2015) examined the effects of a non-uniform heat source/ sink, mass transfer, suction/injection and chemical reaction on MHD unsteady mixed-convection boundary layer flow of a Micropolar fluid over a stretching/shrinking surface. Chamkha (2013) investigated the Soret and chemical reaction effects on mixed-convection MHD

flow past a vertical plate with suction over a porous medium in the presence of heat absorption.

Motivated by the above investigations, this present paper deals with numerical solutions of magnetohydrodynamic mixed convection flow of second grade fluid over a nonlinear stretching surface in presence of the suction or blowing. The process of heat transfer characteristics is examined in the presence of viscous dissipation and internal heat generation/absorption. The considered flow analysis has relevance with the metallurgical process. Such flows have specific relevance to the cooling of continuous strips and filaments drawn through a quiescent fluid, the purification of molten metals from non-metallic inclusions etc. Further, the control of flow of electrically conducting fluid by electromagnetic force in industry is recognized as a useful tool for handling materials like sodium metals and semi conducting melts. Matlab bvp4c package is used to compute the numerical solutions of velocity and temperature distributions. Influences of various embedded flow parameters on the velocity and temperature are presented and discussed in details.

2.0. Formulation of the Problem

Consider the steady MHD mixed convection boundary layer flow of second grade fluid over an impermeable stretching surface in the presence of heat generation/absorption and viscous dissipation. The fluid is electrically conducting under the influence of an applied magnetic field $B(x) = ax^{2n-1}$ normal to the stretching sheet. The flow takes place at $y \geq 0$, where y is the coordinate measured normal to the stretching surface. The flow is generated due to the stretching of the sheet that emerges out of a slit at $x = y = 0$. Let us assume that the speed at a point on the plate is proportional to the power of its distance from the slit which vary nonlinearly with distance x from the leading edge i.e. $U_w(x) = cx^n$, where c is a positive constant

and n is nonlinear stretching parameter. The stretching surface is maintained at prescribed surface temperature $T_w(x) = T_\infty + T_0x^{2n-1}$ (at $y = 0$, where T_0 is a positive constant, n is the surface temperature parameter) higher than the constant temperature T_∞ of the ambient fluid (i.e. temperature of the fluid far away from the surface). Due to the boundary layer behavior, the temperature gradient along y –direction is much more than that along x –direction and hence only the thermophoretic velocity component that is normal to the surface is of significant importance. Under these assumptions, the governing boundary layer equations for a second grade fluid flow can be written as ((Cortell, 2006), (Das et al., 2016))

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \dots\dots\dots (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left(u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} \right) - \frac{\sigma}{\rho} B^2(x)u + g\beta(T - T_\infty) = 0, \dots\dots\dots (2)$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + q(T - T_\infty), \dots(3)$$

here, the velocity components along x, y – axis are u, v respectively, ν is the kinematic viscosity, α_1 is the material fluid parameter, ρ is the fluid density, σ is the electrical conductivity of the fluid, g is the acceleration due to gravity, β is the thermal expansion coefficient, ρc_p is the heat capacitance at constant pressure, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, T and T_∞ are the temperature of the fluid within the boundary layer and the ambient temperature respectively. We assumed that the flow is subjected to suction/blowing on the boundary sheet with velocity $v_w = v_0x^{\frac{n-1}{2}}$. The boundary conditions corresponding to the nonlinear stretching of a sheet problem are:

$$u = U_w(x) = cx^n, \quad v = -v_w, \quad T(x) = T_w(x) = T_\infty + T_0x^{2n-1} \quad \text{at } y = 0,$$

$$u \rightarrow 0, \frac{\partial u}{\partial y} \rightarrow 0, T \rightarrow 0 \quad \text{as } y \rightarrow \infty \dots\dots\dots (4)$$

Where $v_w > 0$ is the suction velocity and $v_w < 0$ is the blowing velocity while $U_w(x)$ is the stretching velocity. Upon making use of the following dimensionless variables:

$$\eta = \sqrt{\frac{(n+1)u}{2vx}} y, \quad \psi(x, y) = \sqrt{\frac{2vux}{(n+1)}} f(\eta), \quad (\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \dots\dots\dots (5)$$

where the stream function $\psi(x, y)$ satisfies the equation (1) with respect to

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \dots\dots\dots (6)$$

the resulting nonlinear ordinary differential system equation using (5) and (6) is simplified as

$$\left(\frac{2n}{n+1}\right) f'^2 - f'''' - f f'' - K \left\{ (3n-1) f' f''' - \left(\frac{n+1}{2}\right) f f^{iv} - \left(\frac{3n-1}{2}\right) f''^2 \right\} + M f' - \lambda \theta = 0, \dots\dots\dots (7)$$

$$\frac{1}{Pr} \theta'' - \left(\frac{2}{n+1}\right) (n f' + H) \theta + f \theta' + Ec f''^2 = 0. \dots\dots\dots (8)$$

The boundary conditions (4) then turn into

$$f = S, f' = 1, \theta = 1 \text{ at } \eta = 0, \\ f' \rightarrow 0, f'' \rightarrow 0, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty. \dots\dots\dots (9)$$

Here prime (') denotes differentiation with respect to η , $K = \frac{\alpha_1 c x^{n-1}}{\mu}$ is the second

grade fluid parameter, $M = \left(\frac{2}{n+1}\right) \frac{\sigma B_0^2}{\rho \alpha}$ is the magnetic field parameter, $\lambda = \frac{g \beta (T_w - T_\infty) x^2}{v U_w}$

is the mixed convection parameter, $S = \left(\frac{2}{n+1}\right)^{\frac{1}{2}} \frac{v_0}{(cv)^{\frac{1}{2}}}$ is the suction/blowing

parameter, $H = \frac{qx}{\rho c_p U_w}$ is the heat generation parameter, $Ec = \frac{U_w^2}{c_p (T - T_\infty)}$ is the Eckert

number and $Pr = \frac{\nu}{\alpha}$ is the Prandtl number. The physical quantities of practical and engineering primary interest are the skin friction coefficient, and Nusselt number. The equation defining the wall shear stress

$$\tau_w = \left[\mu \frac{\partial u}{\partial y} + \rho \alpha_1 \left(u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} - 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \right) \right]_{y=0} \dots\dots\dots (10)$$

The local dimensionless skin friction coefficient is given by

$$C_f = \left(\frac{n+1}{2}\right)^{\frac{1}{2}} Re^{-\frac{1}{2}} \left[1 + K \left(\frac{7n-1}{2}\right) \right] f''(0). \dots\dots\dots (11)$$

Knowing the temperature field, it is interesting to study the effect of the mixed convection on the rate of heat transfer q_w (i.e. heat flux) given by

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \dots\dots\dots (12)$$

So the rate of heat transfer in terms of dimensionless Nusselt number is defined as

$$Nu = -\left(\frac{n+1}{2}\right)^{\frac{1}{2}} Re^{\frac{1}{2}} \theta'(0), \dots\dots\dots (13)$$

where $Re = \frac{u_w x}{\nu}$ is the local Reynolds number.

3.0. Numerical Method for Solution

The system of nonlinear ordinary differential equations (7) – (8) under the boundary conditions (9) forms highly nonlinear-coupled ordinary differential equations. Matlab bvp4c technique is adopted in order to solve these coupled nonlinear differential equations. The step size $\Delta \eta = 0.05$ is used to obtain the numerical solutions with $\eta_{max} = 5$. The procedure is repeated until we get the results up to the desired degree of accuracy, namely 10^{-5} . It should be noted that Matlab bvp4c technique is well established and has been successfully implemented to study a variety of nonlinear fluid flow problems. To check the validity of the numerical code, the values of different values of $-\theta'(0)$ have been calculated for $n = 0.5, H = 0.1$ and $M = \lambda = Pr = Ec = S = 0$ and for different values of second grade fluid parameter as shown in Table 3.1. It has been observed from the table that the results obtained by the present code and that of

Prasad et al. (2010a) show an excellent agreement. Thus, the present numerical code used for current model is justified.

4.0. Results and Discussion

The numerical computations are performed using the method described in the previous section for various values of parameters that describe the flow characteristics of a second grade fluid over a non-isothermal stretching sheet. The results are illustrated graphically in Figures 4.1– 4.10 and in Table 4.2. There are many parameters involved in the final form of the model. The problem can be protracted on many directions, from the effects of nonlinear stretching sheet parameter n , second grade fluid parameter K , magnetic field parameter M , mixed convection parameter λ , Prandtl number Pr , heat generation/absorption parameter H , Eckert number Ec and suction/blowing parameter S . The default values of material parameters are considered in the simulation as $n = M = 2, K = H = 0.5, \lambda = 3, Pr = 7, Ec = 0.3$ and $S = 1$ unless otherwise specified.

Table 3.1: Comparison of Skin friction coefficients $-f''(0)$ for various values of second grade fluid parameter when $n = 0.5, H = 0.1$ and $M = \lambda = Pr = Ec = S = 0$.

K	Prasad et al. (2010a)	Present result
0.1	1.0663694	1.066360
0.2	1.1113553	1.111351
0.3	1.1909626	1.190969

The effect of heat generation/absorption parameter H on the temperature profile is presented in Figure 4.1. The results presented in this figure indicates that the magnitude of the temperature profile increases as H increases from -3 to 3 . The positive values of $H > 0$ refers to the heat generation and negative values of $H < 0$ refers to the heat absorption. It is therefore observed from the figure that in the presence of a heat generation effect ($H > 0$), the

thermal state of the fluid increases. Hence, the temperature of the fluid increase within the nonlinear stretching boundary layers. In the situation that the strength of the heat generation is relatively large, the overshoot is observed in the temperature profile within the nonlinearly stretching thermal boundary layers as can be seen in the figure. On the other hand, the presence of heat absorption effect ($H < 0$) has the tendency to reduce the temperature of fluid. This causes the thermal buoyancy effects to decrease resulting in a net reduction in the fluid velocity. It can be seen from Table 4.1 that the magnitude of the skin friction coefficient increases whereas the heat transfer rate decreases at the wall in the presence of a heat generation ($H > 0$) and increases monotonously in the presence of heat absorption ($H < 0$). This is evident from the fact that negative values of $H (< 0)$ would cause an enhancement of temperature difference between the moving surface and the convective fluid, and thus decrease the heat transfer rate. It is also important to note that the heat transfer rate is at steady state when there is no effect of heat generation or absorption $H = 0$.

Figure 4.2 illustrates the effect of Eckert number Ec on the temperature profile. The effects of viscous dissipation parameter of the fluid heating case ($Ec = 0.3$), exhibits that temperature profile are smaller than that of the case without viscous dissipation ($Ec = 0$), whereas the opposite trend is observed for the fluid cooling case ($Ec = -0.3$). The overshoot of temperature profile increases with viscous dissipation parameter ($Ec = -0.3$). It is found from Table 4.1 that heat transfer rate decreases in case of fluid heating $Ec = 0.3$ as compared to the case of fluid cooling $Ec = -0.3$.

Figure 4.3 depict the effect of suction or blowing parameter on temperature profile. The role of blowing is that the fluid is carried away from the nonlinearly stretching surface, which reduces the velocity gradient as it tries to maintain the same velocity level

over a very small vicinity of the nonlinearly stretching surface. The effect is quite reversed in case of suction. The higher magnitude overshoot is observed near the wall within the boundary layer for blowing ($S < 0$) and overshoot is decreased for suction ($S > 0$). Blowing takes a role in reducing the steepness of temperature profile where steepness of temperature increases with suction. It is also observed from Table 4.1 that, the skin friction coefficient and heat transfer rate increase with the increase in suction ($S > 0$) or blowing ($S < 0$) parameter. When suction or blowing parameter S decreases from -1 to 1 , the heat transfer rate as well as the skin friction coefficient decreases.

The variation of Prandtl number Pr on dimensionless temperature profile is plotted in Figure 4.4. With increasing values of Pr , the fluid temperature and thermal boundary layer thickness is found to be decreasing notably. This is due to the fact that, the Prandtl number Pr is defined as the ratio of momentum diffusivity to thermal diffusivity. Thermal diffusivity of the fluid reduces with an increase in Prandtl number, and this leads to decrease in temperature profile. It is noted that, fluids with lower Pr will possess higher thermal conductivities so that heat can diffuse from the sheet faster than for higher Pr fluids.

Figures 4.5 – 4.6 show the effects of mixed convection parameter λ on velocity and temperature profiles for $n = 1$ and $n = 2$. When $n = 1$, results correspond to linear stretching surface and for $n = 2$ results correspond to nonlinearly stretching surface. The influence of buoyancy assisting force ($\lambda > 0$) displays the overshoot in the velocity profile. The magnitude of velocity overshoots increases with the increase of buoyancy parameter ($\lambda > 0$). The physical reason is that buoyancy assisting force ($\lambda > 0$) due to thermal gradients and heat generation combined acts like a favourable pressure gradient, in turns, which enhances the fluid

acceleration and increases the magnitude of velocity profile within the nonlinear stretching boundary layer. It is important to note that the buoyancy opposing force ($\lambda < 0$) reduces the magnitude of the velocity considerably within the nonlinear stretching boundary layer.

Comparative studies on $n = 1$ and $n = 2$ indicate that the magnitude of the velocity overshoot decreases remarkably when $n = 2$, i.e. nonlinearly stretching surface within the stretching boundary layer as compared to the linear stretching surface case ($n = 1$). It is found from Table 4.1 that the variation of skin friction coefficients observed to increases near the surface as the value of mixed convection parameter λ increases. The physical reason is that the buoyancy force ($\lambda > 0$) implies favourable pressure gradient, and the fluid gets accelerated, which results in thinner momentum and thermal boundary layers.

The effects of magnetic field parameter M on the velocity and temperature profiles are presented in Figures 4.7 and 4.8. These figures disclosed that, the velocity field decreases and temperature distribution increases with an increase in the strength of magnetic field. Since the effect of magnetic field on an electrically conducting fluid results a resistive type of force called Lorentz force, which has tendency to decrease the fluid velocity and to increase the temperature field. Due to this fact, magnetic field effect has many possible control-based applications like MHD ion propulsion, electromagnetic casting of metals, in MHD power generation etc.

The effect of second grade parameter K on the fluid velocity and temperature distribution is illustrated in Figures 4.9 and 4.10. One may see from Figure 4.9 that the velocity component across the boundary layer intensify with an increase in the second grade parameter and also rises asymptotically to free stream at the far of the hydrodynamic boundary layer. From

Figure 4.10 it is noticed that with increase in the second grade parameter, the temperature profiles decreases and hence thickness of thermal boundary layer increases. From Table 4.1, it is observed that with increasing the values of K , the reduced skin friction coefficient (in absolute sense) increases whereas the reduced Nusselt number diminishes. We see from this figure that, the

magnitude of velocity is larger in the case of non-Newtonian viscoelastic fluid ($K > 0$) compared to the Newtonian fluid ($K = 0$). It is recognized that the magnitude of the local skin friction coefficient exhibits increasing trend for the enlarging values of the generalized second.

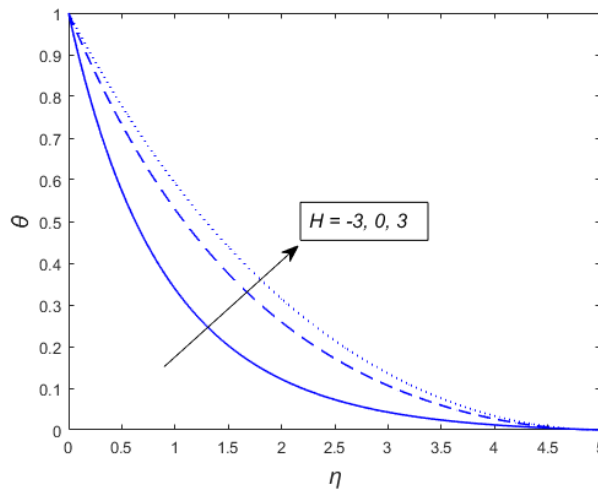


Figure 4.1: Effect of the generalized heat generation/absorption parameter on temperature profile.

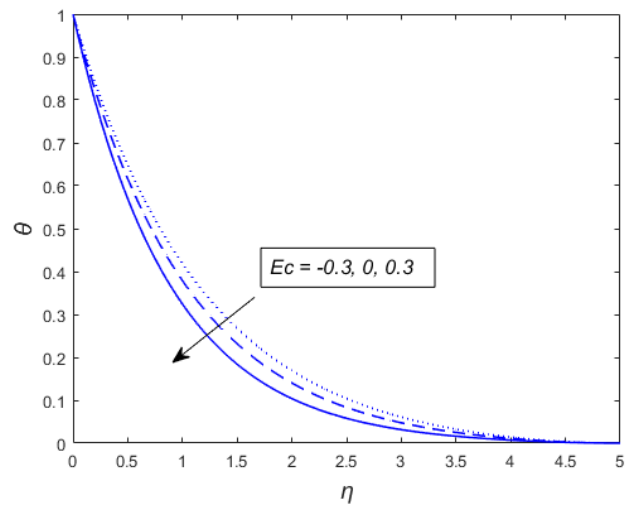


Figure 4.2: Effect of the generalized Eckert number on temperature profile.

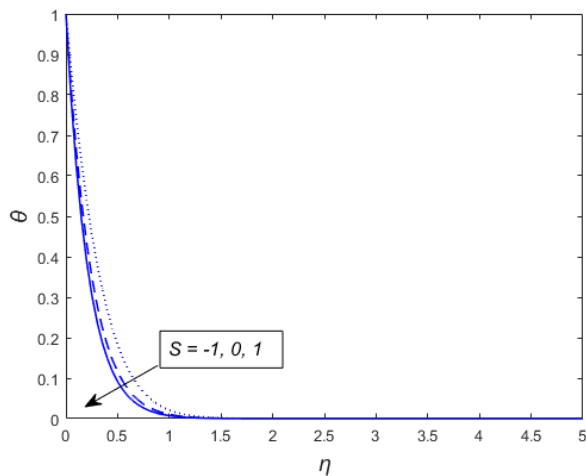


Figure 4.3: Effect of the generalized suction/blowing parameter on temperature profile.

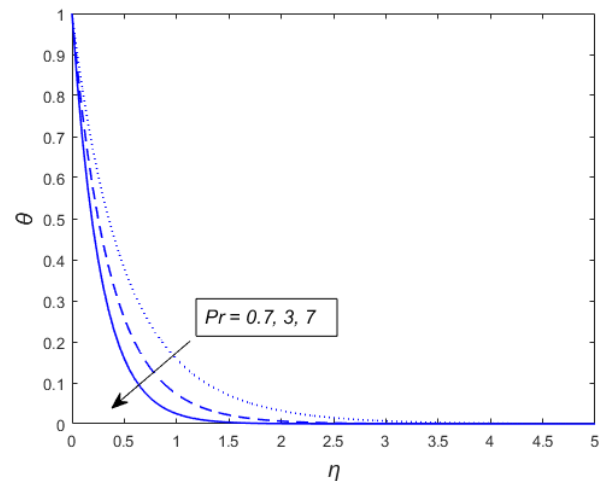


Figure 4.4: Effect of the generalized Prandtl number on temperature profile.

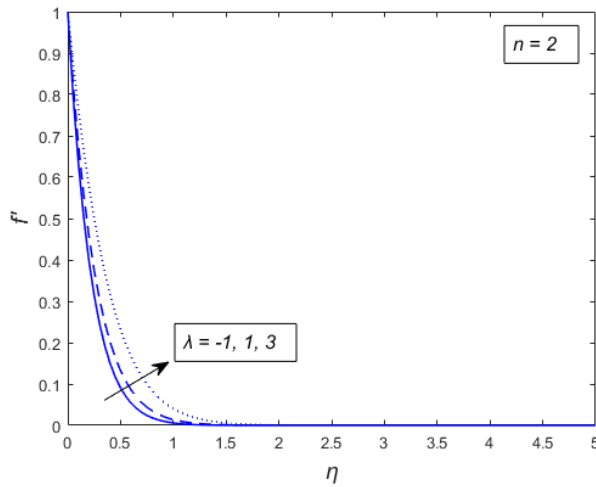


Figure 4.5: Effect of the generalized mixed convection parameter on velocity profile.

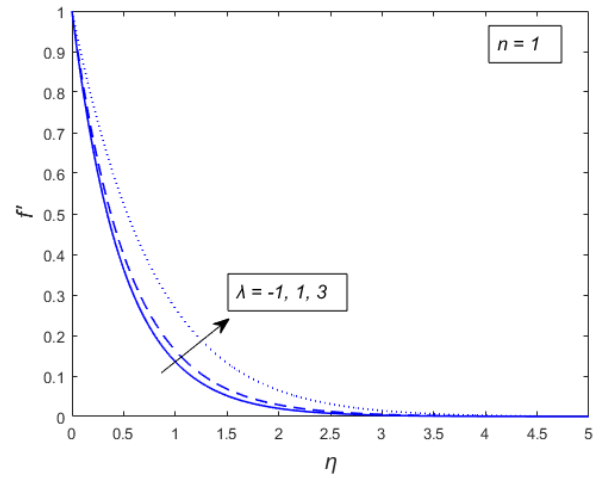


Figure 4.6: Effect of the generalized mixed convection parameter on velocity profile.

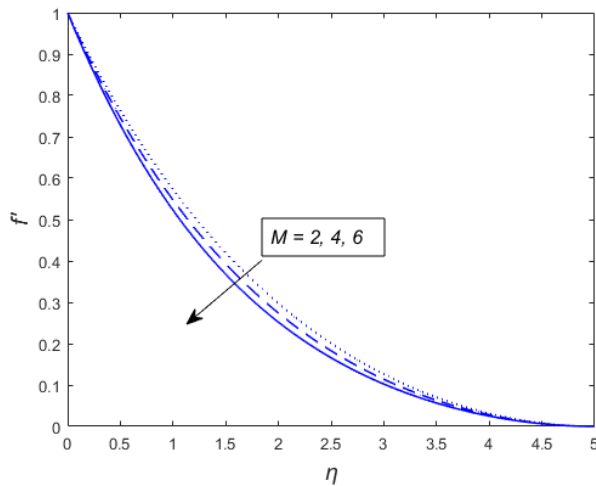


Figure 4.7: Effect of the generalized magnetic field parameter on velocity profile.

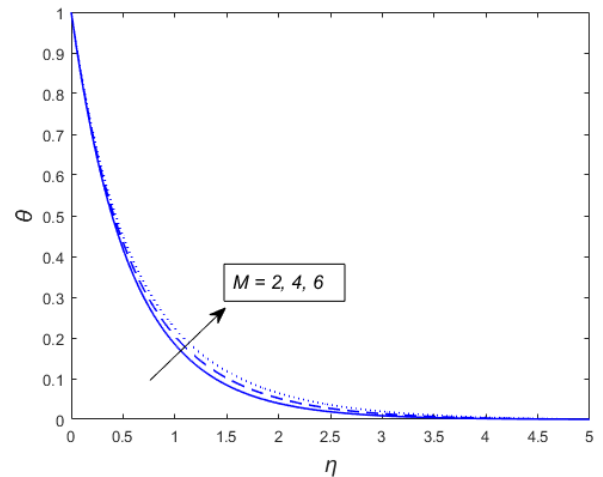


Figure 4.8: Effect of the generalized magnetic field parameter on temperature profile.

Table 4.1: Numerical results of skin friction coefficient $f''(0)$ and heat transfer rate $\theta'(0)$ for various value of λ, K, S, Ec and H with $Pr = 3.0, n = M = 2.0$,

λ	K	S	Ec	H	$f''(0)$	$-\theta'(0)$
-1.0	2.0	1.0	0.3	3.0	-4.08340	1.59459
1.0	2.0	1.0	0.3	3.0	-1.76617	1.70671
3.0	2.0	1.0	0.3	3.0	-2.29633	1.82259
3.0	2.0	1.0	0.3	3.0	-5.55354	0.07891
3.0	4.0	1.0	0.3	3.0	-9.14144	0.06629
3.0	6.0	1.0	0.3	3.0	-12.61510	0.05479
3.0	2.0	-1.0	0.3	3.0	-0.82598	1.37531
3.0	2.0	0.0	0.3	3.0	-0.83136	1.68386
3.0	2.0	1.0	0.3	3.0	-0.79055	2.05515
3.0	2.0	1.0	-0.3	3.0	-1.75022	2.91500
3.0	2.0	1.0	0.0	3.0	-1.72452	2.83822
3.0	2.0	1.0	0.3	3.0	-1.69822	2.76329
3.0	2.0	1.0	0.3	-3.0	-4.19394	3.43639
3.0	2.0	1.0	0.3	0.0	-4.21383	3.27124
3.0	2.0	1.0	0.3	3.0	-4.23610	3.09467

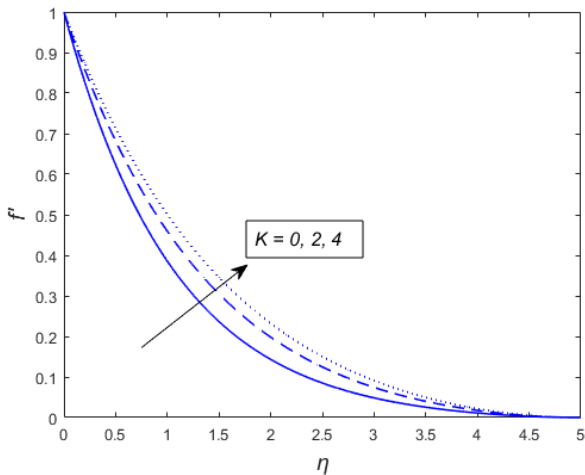


Figure 4.9: Effect of the second grade fluid parameter on velocity profile.

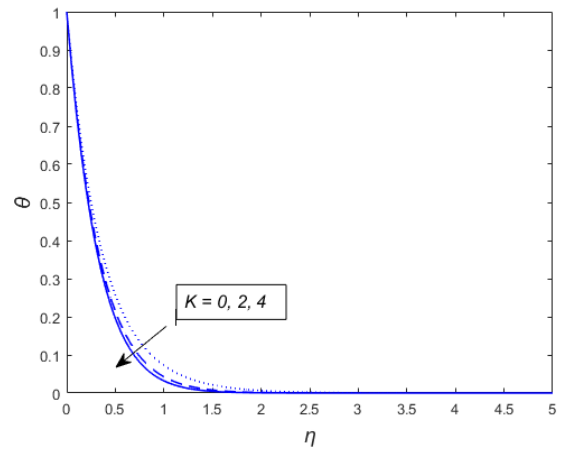


Figure 4.10: Effect of the second grade fluid parameter on temperature profile.

5.0. Conclusion

A numerical investigation to study a steady magnetohydrodynamics boundary layer flow and heat transfer of second grade fluid towards nonlinearly stretching surface in the presence of suction and blowing is considered with the combined effects of buoyancy force, heat generation/absorption and viscous dissipation. The governing equations are transformed into a system of ordinary differential equations by applying suitable similarity transformations and then solved numerically using bvp4c Matlab package. The results are presented through graphs and tables to illustrate the details of the flow characteristics and their dependence on material parameters. Numerical results indicate that the buoyancy force (mixed convection parameter), suction or blowing enhanced the magnitude of the skin friction coefficient and heat transfer rate. The fluid velocity in the boundary layer region increases for increasing the values second grade parameter, and the rate of heat transfer decreases with increase in the values of second grade parameter. Furthermore, it is noted that the suction parameter ($S > 0$) is to strengthening the velocity and temperature profiles, but blowing parameter ($S < 0$) does the reverse. It is also noted that, fluids with lower Pr will possess higher thermal conductivities so that heat can diffuse from the sheet faster than for higher Pr fluids.

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